hep-ph/9609314
TRI–PP–96–49
September 1996

Supersymmetric Time Reversal Violation in Semileptonic Decays of Charged Mesons

Guo-Hong Wu * and John N. Ng †

TRIUMF Theory Group

4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A3

Abstract

We provide a general analysis of time reversal violation arising from misalignment between quark and squark mass eigenstates. In particular, we focus on the possibility of large enhancement effects due to the top quark mass. For semileptonic decays of the charged mesons, $K^+ \to \pi^0\mu^+\nu_\mu$, $D^+ \to \overline{K}^0\mu^+\nu_\mu$, and $B^+ \to \overline{D}^0\tau^+\nu_\tau$, the transverse polarization of the lepton $P_l^\perp$ is a T-odd observable that is of great experimental interest. It is noted that under favorable choice of parameters, $P_\mu^\perp$ in $K_{\mu3}^+$ decay can be detectable at the ongoing KEK experiment and it holds a promising prospect for discovery at the proposed BNL experiment. Furthermore, $P_\tau^\perp$ in $B^\pm$ decay could well be within the reach of B factories, but $P_\mu^\perp$ in $D^\pm$ decay is not large enough for detection at the proposed τ-charm factory.

*gwu@alph02.triumf.ca
†misery@triumf.ca

In supersymmetric (SUSY) theories, different unitary transformations are generally required for the quark and squark gauge eigenstates to reach their respective mass eigenstates [1]. The difference between the quark and squark transformations is referred to as quark-squark misalignment (QSM), a consequence of which is the appearance of nontrivial family mixing matrices in the gluino–quark–squark couplings and thus new contributions to flavor changing neutral current (FCNC) processes arise. The physical phases in these mixing matrices could serve as new sources of hadronic CP and T violation (assuming CPT invariance), and a similar mechanism can operate in the lepton sector. In this context, the electric dipole moments of the neutron and the electron have recently been studied in the minimal supersymmetric standard model (MSSM) [2], the minimal SUSY $SO(10)$ [3], and SUSY grand unified models with an intermediate scale [4].

In this letter, we study a different manifestation of QSM, i.e. its effects on the T-odd transverse lepton polarization in semileptonic decays of charged mesons, $K^+ \to \pi^0 \mu^+ \nu_\mu$, $D^+ \to \overline{K}^0 \mu^+ \nu_\mu$, and $B^+ \to \overline{D}^0 \tau^+ \nu_\tau$. This observable measures the T-odd triple correlation among the spin of the charged lepton and the momenta of two final state particles. In the rest frame of the decay meson, it is defined as [5]

$$P_l^\perp \equiv \frac{\hat{\mathbf{s}}_\mathbf{l} \cdot (\mathbf{n_M} \times \mathbf{n_l})}{|\mathbf{n_M} \times \mathbf{n_l}|}, \tag{1}$$

where $\hat{\mathbf{s}}_\mathbf{l}$ is the unit vector along the the charged lepton spin direction, and $\mathbf{n_M}$ and $\mathbf{n_l}$ are unit vectors along the three momenta of the final state meson and the charged lepton respectively. For the charge conjugate processes of the above decays, CP-violating interactions contribute to $P_l^\perp$ with an opposite sign [6].

A nonzero $P_l^\perp$ arises from the interference between two amplitudes with different phases. In this case it involves the standard model weak decay amplitude and an effective scalar amplitude [7]. As there is only one charged particle in the final state, the electromagnetic final state interaction contribution to $P_l^\perp$ is expected to be small, of order 10^{-6} for $K^+_{\mu 3}$ decay [8]. The standard model T-violation effect is vanishingly small [9], leaving enough

room for new physics to come in. For example, in the Weinberg model of CP violation [10], where $P_l^\perp$ arises from the interference between two tree level amplitudes, an effect as large as 10^{-3} can be obtained for the transverse muon polarization in $K^+_{\mu 3}$ decay [11], and much larger contributions are possible for the transverse τ polarization in $B^+ \to \overline{D}^0 \tau^+ \nu_\tau$ [12–14]. In the MSSM, if one assumes that T violation comes from the complex soft SUSY breaking operators and neglects squark family mixing, $P_l^\perp$ will be suppressed by the strange quark mass m_s for $K^+_{\mu 3}$, the charm quark mass m_c for $D^+_{\mu 3}$, and the b quark mass m_b for $B^+_{\tau 3}$, and would be too small to to be seen [15].

With the squark family mixing taken into account, $P_l^\perp$ in the semileptonic decays of charged kaon, D and B mesons could become sensitive to quarks of the third family, in particular to the top quark. This comes about in two ways. Firstly, both the $\tilde{t}_L$–$\tilde{t}_R$ stop mixing and the charged Higgs coupling to $\tilde{t}_R$ have strengths proportional to the top mass m_t. Secondly, due to the renormalization group equation (RGE) running from a higher scale, the third generation squarks tend to be lighter at the weak scale than squarks of the first two generations because of the large top Yukawa coupling. These lead to the possibility that the $\tilde{t}$ and $\tilde{b}$ squarks could give the largest contribution to $P_l^\perp$. This possibility will be the focus of this letter.

The various squark family mixing matrices can be determined in specific models, however for the sake of generality we will not be restricted to a particular one but rather give a general description. We refer interested readers to the literature [16,17] for a discussion on models. The mixing matrices characterizing the relative rotations in flavor space between the four types of squarks ($\tilde{u}_L$, $\tilde{u}_R$, $\tilde{d}_L$ and $\tilde{d}_R$) and their corresponding quarks are denoted by V^{U_L}, V^{U_R}, V^{D_L}, and V^{D_R} respectively. These matrices appear in the quark-squark-gluino couplings, and are constrained by FCNC processes [1]. These constraints can be divided into two classes. Firstly, products of V^D's are bounded by FCNC in the down quark sector, using $K\overline{K}$ and $B\overline{B}$ mixing, $b \to s\gamma$, ϵ_K, and ϵ'_K. Secondly, products of V^U's are bounded by FCNC in the up quark sector, and $D\overline{D}$ mixing provides the only available constraint. Note that $D\overline{D}$

mixing can put nontrivial bounds on products of $V_{32}^{U_{L,R}}$ and $V_{31}^{U_{L,R}}$, but not on the individual matrix element. Therefore large mixing in 32 or 31 up type squark families is not ruled out [18]. We also note that the neutron electric dipole moment can constrain a particular combination of phases and product of two different squark generational mixings, but not the individual mixing matrix element. In contrast to FCNC processes, SUSY contributions to charged current processes, including the semileptonic decays of charged mesons, can involve products of V^U and V^D that are not subject to direct constraints from FCNC. By invoking the mixing matrix V^{U_R} in the loop, these charged meson decays can be enhanced by the top quark mass. The usual KM matrix is denoted as V^{KM}.

The physical phases that are responsible for T-violation can arise from the squark family mixing matrices and other soft SUSY breaking operators including the gaugino masses and the A terms. Typically, T-odd effects involve the difference of two or more physical phases. For simplicity of illustration of the physics and for an estimate of the T-odd effects, only the phases from the squark mixing matrices will be kept. Throughout this letter, we will neglect the mixing between the left and right squarks except for the top squarks, as we expect them to play a more important role. Also we will employ an insertion approximation for the $\tilde{t}_L$–$\tilde{t}_R$ mixing where appropriate.

The dominant contribution to $P_l^\perp$ is thus expected to come from the gluino-$\tilde{t}$-$\tilde{b}$ loop diagrams with W and charged Higgs boson exchange (figures 1 and 2). To linear order in the external momenta and to first order in the $\tilde{t}_L$–$\tilde{t}_R$ mixing, the W exchange diagram (fig. 1) for kaon decay can be evaluated to give

$$\mathcal{L}_{eff,W} = [C_W^+(p+p')^\alpha + C_W^-(p-p')^\alpha](\overline{s}_L u_R)(\overline{\nu}_L \gamma_\alpha \mu_L) + h.c., \tag{2}$$

where p and p' are the momenta of the s and u quarks respectively, and $C_W^\pm$ are given by

$$C_W^\pm = \frac{1}{18}\frac{\alpha_s}{\pi}\sqrt{2}G_F \frac{m_t(A_t - \mu\cot\beta)}{m_{\tilde{g}}^3} V_{33}^{SKM*} V_{32}^{D_L *} V_{31}^{U_R} I_W^\pm, \tag{3}$$

where G_F is the Fermi constant, α_s is the QCD coupling evaluated at the mass scale of the sparticles in the loop, A_t is the soft SUSY breaking A term for the top squarks, μ denotes

the two Higgs superfields mixing parameter, $\tan\beta$ is the ratio of the two Higgs VEVs, $m_{\tilde{g}}$ is the mass of the gluino, and V_{ij}^{SKM} is the super KM matrix associated with the W-squark coupling $W^+\tilde{u}_{iL}^*\tilde{d}_{jL}$. Assuming $m_{\tilde{t}_L} = m_{\tilde{t}_R} = m_{\tilde{t}}$ for the mass parameters of the left and right top squarks and denoting the bottom squark mass by $m_{\tilde{b}}$, the integrals I_W^+ and I_W^- can be written as

$$I_W^+ = \int_0^1 dx \int_0^{1-x} dy \frac{24x(1-x-y)}{[\frac{m_{\tilde{t}}^2}{m_{\tilde{g}}^2}x + \frac{m_{\tilde{b}}^2}{m_{\tilde{g}}^2}y + (1-x-y)]^2} \tag{4}$$

$$I_W^- = \int_0^1 dx \int_0^{1-x} dy \frac{24x(x-y)}{[\frac{m_{\tilde{t}}^2}{m_{\tilde{g}}^2}x + \frac{m_{\tilde{b}}^2}{m_{\tilde{g}}^2}y + (1-x-y)]^2}. \tag{5}$$

Both integrals $I_W^{\pm}$ are equal to one at $\frac{m_{\tilde{t}}}{m_{\tilde{g}}} = \frac{m_{\tilde{b}}}{m_{\tilde{g}}} = 1$, and both increase as $\frac{m_{\tilde{t}}}{m_{\tilde{g}}}$ and/or $\frac{m_{\tilde{b}}}{m_{\tilde{g}}}$ decreases from one. For example, $I_W^- \simeq 7.9$ when $\frac{m_{\tilde{t}}}{m_{\tilde{g}}} = \frac{m_{\tilde{b}}}{m_{\tilde{g}}} = \frac{1}{2}$.

Notice that the C_W^- term in eq. (2) can be rewritten as an effective scalar interaction by use of the Dirac equation for the external leptons. The term $(p+p')^\alpha(\overline{s}_L u_R)$ in eq. (2) can be Gordon decomposed into a tensor piece and a vector piece. The tensor piece can be neglected as the tensor form factor is small. The vector piece, having the same structure as the standard model interaction, is readily discarded as its contribution to $P_l^\perp$ vanishes [7]. Therefore, T-violating effects arise dominantly from the interference between the effective scalar amplitude of the C_W^- term in eq. (2) and the effective current-current amplitude of the standard model.

To proceed further, we parameterize the hadronic matrix elements of the quark vector current and quark scalar density between a kaon and a pion state by two form factors

$$\langle \pi^0 | \overline{s}\gamma_\mu u | K^+ \rangle = f_+^K (p_K + p_\pi)_\mu + f_-^K (p_K - p_\pi)_\mu \tag{6}$$

$$\langle \pi^0 | \overline{s} u | K^+ \rangle \simeq -\frac{f_+^K m_K^2}{m_s}, \tag{7}$$

where p_K and p_π denote the kaon and pion momenta, and where the two form factors f_+^K and f_-^K are functions of $(p_K - p_\pi)^2$. Experimentally, f_-^K is small compared to f_+^K, and its contribution to the scalar matrix element in eq. (7) has been neglected to a first approximation.

The lepton transverse polarization can now be estimated following the standard procedure [11]. In the kaon rest frame, for the outgoing muon and neutrino coming at right angle for which $P_\mu^\perp$ is large, the W-exchange contribution gives

$$\begin{aligned} P_\mu^\perp|^W_{\mathbf{n}_\mu\cdot\mathbf{n}_\nu=\mathbf{0}} &\simeq (2\sqrt{2}G_F\sin\theta_c)^{-1}\frac{m_\mu m_K}{m_s}\frac{|\mathbf{p}_\mu|}{E_\mu}ImC_W^- \\ &\simeq \frac{\alpha_s}{36\pi}I_W^-\frac{|\mathbf{p}_\mu|}{E_\mu}\frac{m_\mu m_K}{m_s}\frac{m_t(A_t-\mu\cot\beta)}{m_{\tilde g}^3}\frac{Im[V_{33}^{SKM*}V_{32}^{D_L*}V_{31}^{U_R}]}{\sin\theta_c} \\ &\simeq 3\times 10^{-6}I_W^-\frac{|\mathbf{p}_\mu|}{E_\mu}\frac{m_t(A_t-\mu\cot\beta)}{m_{\tilde g}^2}\frac{100\text{ GeV}}{m_{\tilde g}}\frac{Im[V_{33}^{SKM*}V_{32}^{D_L*}V_{31}^{U_R}]}{\sin\theta_c} \end{aligned} \tag{8}$$

where we take $\alpha_s \simeq 0.1$, θ_c is the Cabibo angle, m_μ, m_K and $m_s \simeq 150$ MeV denote the masses of the muon, the charged kaon and the strange quark, and where $\mathbf{p}_\mu$ and E_μ are the outgoing muon momentum and energy.

It is readily seen from eq. (8) that the actual size of $P_\mu^\perp$ from W exchange is model dependent in two aspects. Firstly, it depends on the mass spectrum of the SUSY particles, in particular on $m_{\tilde g}$ and on I_W^- through $m_{\tilde t}/m_{\tilde g}$ and $m_{\tilde b}/m_{\tilde g}$. As noted before from eq. (5), $P_\mu^\perp$ can be enhanced by one order of magnitude if $m_{\tilde t}$ and $m_{\tilde b}$ decrease from the mass of the gluino to half of its mass. Secondly, it is dependent on the mixing matrices V_{33}^{SKM}, $V_{32}^{D_L}$ and $V_{31}^{U_R}$. As these parameters are strictly unknown and the bounds on them are model dependent, the number in eq. (8) should be taken as qualitative. For an estimate of the effect, the dimensionful SUSY parameters appearing in eq. (8) can be set to be 100 GeV. With these caveats, we conclude that the magnitude of $P_\mu^\perp$ from W exchange is no larger than a few $\times 10^{-5}$.

We now consider the contribution from charged Higgs exchange which can involve several diagrams. As we are interested in the largest possible effect from the top quark, it can be seen that the dominant contribution involves the $H^-\tilde t_R\tilde b_L^*$ coupling and a gluino in the loop (fig. 2). The effective interaction obtained by integrating out the $\tilde t_R$, $\tilde b_L$, $\tilde g$ and H^+ is given by

$$\mathcal{L}_{eff,H^+} = \frac{C_{H^+}}{m_{H^+}^2}(\overline{s}_L u_R)(\overline{\nu}_L\mu_R) + h.c., \tag{9}$$

where m_{H^+} is the mass of the charged Higgs boson, and C_{H^+} is defined as

$$C_{H^+} = -\frac{2}{3}\frac{\alpha_s}{\pi}\sqrt{2}G_F m_t m_\mu \tan\beta \frac{\mu + A_t \cot\beta}{m_{\tilde{g}}} V_{33}^{H^+*} V_{32}^{D_L*} V_{31}^{U_R} I_{H^+}, \tag{10}$$

where $V_{ij}^{H^+}$ is the mixing matrix in the charged-Higgs-squark coupling $H^+ \tilde{u}_{iR}^* \tilde{d}_{jL}$, and the integral function I_{H^+} is given by

$$I_{H^+} = \int_0^1 dx \int_0^{1-x} dy \frac{2}{\frac{m_{\tilde{t}}^2}{m_{\tilde{g}}^2}x + \frac{m_{\tilde{b}}^2}{m_{\tilde{g}}^2}y + (1-x-y)}. \tag{11}$$

which is equal to one at $m_{\tilde{t}} = m_{\tilde{b}} = m_{\tilde{g}}$. In the above integral $m_{\tilde{t}}$ and $m_{\tilde{b}}$ denote the mass parameters of $\tilde{t}_R$ and $\tilde{b}_L$. The size of I_{H^+} increases as $m_{\tilde{t}}/m_{\tilde{g}}$ and/or $m_{\tilde{b}}/m_{\tilde{g}}$ decreases, but not as rapidly as the integral function I_W^-. For example, I_{H^+} increases to 2.3 as $m_{\tilde{t}}$ and $m_{\tilde{b}}$ decrease to half the gluino mass.

In the kaon rest frame and for $\mathbf{n}_\mu \cdot \mathbf{n}_\nu = \mathbf{0}$, the muon transverse polarization from charged Higgs exchange can be estimated as

$$\begin{aligned} P_\mu^\perp|_{\mathbf{n}_\mu\cdot\mathbf{n}_\nu=\mathbf{0}}^{H^+} &\simeq -(2\sqrt{2}G_F m_{H^+}^2 \sin\theta_c)^{-1}\frac{m_K}{m_s}\frac{|\mathbf{p}_\mu|}{E_\mu} Im C_{H^+} \\ &\simeq \frac{\alpha_s}{3\pi} I_{H^+}\frac{|\mathbf{p}_\mu|}{E_\mu}\frac{m_K}{m_s}\frac{m_t m_\mu}{m_{H^+}^2}\tan\beta\frac{(\mu + A_t\cot\beta)}{m_{\tilde{g}}}\frac{Im[V_{33}^{H^+*}V_{32}^{D_L*}V_{31}^{U_R}]}{\sin\theta_c} \\ &\simeq 7\times 10^{-5} I_{H^+}\frac{|\mathbf{p}_\mu|}{E_\mu}\tan\beta\frac{\mu + A_t\cot\beta}{m_{\tilde{g}}}\frac{(100\text{ GeV})^2}{m_{H^+}^2}\frac{Im[V_{33}^{H^+*}V_{32}^{D_L*}V_{31}^{U_R}]}{\sin\theta_c}, \end{aligned} \tag{12}$$

where we use $\alpha_s \simeq 0.1$ and $m_t = 180$ GeV. Note that there are two crucial differences between the Higgs contribution of eq. (12) and the W contribution of eq. (8). Firstly, due to the non-derivative coupling of the charged Higgs to the squarks (in contrast to the derivative coupling for the W-squark-squark vertex), the numerical coefficient in eq. (12) is one order of magnitude bigger than that of eq. (8). Secondly, unlike the W exchange, charged Higgs coupling to $\overline{\nu}\mu$ is proportional to $\tan\beta$, and its contribution to $P_\mu^\perp$ can be greatly enhanced when $\tan\beta$ is large. To date the best limit on $\tan\beta$ and m_{H^+} comes from $b \to c\tau\nu$ and is given by $\frac{\tan\beta}{m_{H^+}} < 0.52\text{ GeV}^{-1}$ [19]. Therefore Higgs exchange effect can be larger than W exchange by two to three orders of magnitude.

Like the W contribution, charged Higgs contribution to $P_\mu^\perp$ also depends on the squark mixings $|V_{32}^{D_L}|$ and $|V_{31}^{U_R}|$ (taking $|V_{33}^{H^+}| \sim 1$). As pointed out earlier, V^U's are constrained by $D\overline{D}$ mixing only in the product of $V_{31}^{U_L,R}$ and $V_{32}^{U_L,R}$, and $|V_{31}^{U_R}| \sim \mathcal{O}(1)$ is still allowed. On the other hand, assuming $|V_{33}^{D}| \sim \mathcal{O}(1)$, the FCNC process $b \to s\gamma$ can put a bound on $|V_{32}^{D_L}|$ from the gluino-$\tilde{b}$ diagram. However, other SUSY contributions to $b \to s\gamma$, including the charged Higgs and chargino contributions [20], can dominate over the gluino effect and render the bound on $|V_{32}^{D_L}|$ meaningless. This is particularly true if the chargino is relatively light. To estimate the upper limit on $P_\mu^\perp$ from Higgs exchange, we therefore assume maximal squark mixings with $|V_{32}^{D_L}| = |V_{31}^{U_R}| = 1/\sqrt{2}$ and take $m_{H^+} = 100$ GeV and $\tan\beta = 50$. Setting $|\mu| = A_t = m_{\tilde{g}}$ and $I_{H^+} = 1$, we get for the magnitude of $P_\mu^\perp$ in $K_{\mu 3}^+$ decay

$$P_\mu^\perp|_{\mathbf{n}_\mu\cdot\mathbf{n}_\nu=\mathbf{0}}^{K^+\text{ decay}} < 7 \times 10^{-3}, \tag{13}$$

where the upper bound in eq. (13) is for the kinematically allowed maximal value of $\frac{|\mathbf{p}_\mu|}{E_\mu} \simeq 0.90$. In the absence of squark family mixing, $P_\mu^\perp$ will be suppressed relative to the estimate given above by $\frac{m_s V_{us}^{KM}}{m_t} \sim 2 \times 10^{-4}$.

We note in passing that in the large $\tan\beta$ limit, $P_l^\perp$ due to Higgs exchange involving the $H^-\tilde{t}_L\tilde{b}_R^*$ coupling may not be negligible in comparison to that involving the $H^-\tilde{t}_R\tilde{b}_L^*$ coupling considered above. However, the upper limit given by eq. (13) is not expected to be significantly modified. The same can be said for D and B semileptonic decays to be discussed below.

The present experimental limit on the transverse muon polarization in $K_{\mu 3}^+$ decay was obtained fifteen years ago at the BNL-AGS. The combined value is $P_\mu^\perp = (-1.85 \pm 3.60) \times 10^{-3}$ [21], and this implies $|P_\mu^\perp| < 0.9\%$ at the 95% confidence level. The on-going KEK E246 experiment [22] is aimed to reach a sensitivity of 9×10^{-4}. More recently, studies on the BNL-AGS experiments show that to measure $P_\mu^\perp$ in $K_{\mu 3}^+$ to an accuracy of $\sim 10^{-4}$ can be done [23], and to achieve a higher precision of 10^{-5} is not impossible [24]. The above

analysis suggests that whereas the KEK experiment is important in testing the SUSY T violation, higher precision measurements at the AGS are required to pin down the SUSY parameter space, including moderate squark inter-family mixings and the low $\tan\beta$ region.

A similar analysis can be done for the semileptonic decays of the charged D and B mesons. However, we will concentrate on the latter in light that B factories will soon be operating. We consider $B^+ \to \overline{D}^0 \tau^+ \nu_\tau$ to take advantage of the larger τ lepton mass. The W exchange (see fig. 1 with the external fermions properly replaced) gives rise to an effective interaction similar to eq. (2). In the B rest frame, we get

$$\begin{aligned} P_\tau^\perp|^W_{\mathbf{n}_\tau\cdot\mathbf{n}_\nu=\mathbf{0}} &\simeq \frac{\alpha_s}{36\pi} I_W^- \frac{|\mathbf{p}_\tau|}{E_\tau} \frac{m_t(A_t-\mu\cot\beta)}{m_{\tilde g}^3} \frac{m_\tau m_B}{m_b-m_c} \left(1+\frac{f_-^B(p_B-p_D)^2}{f_+^B m_B^2}\right)\times \\ &\qquad \times \frac{Im[V_{33}^{SKM*} V_{33}^{D_L*} V_{32}^{U_R} V_{cb}^{KM}]}{|V_{cb}^{KM}|^2} \\ &\simeq 3\times 10^{-5} I_W^- \frac{|\mathbf{p}_\tau|}{E_\tau} \frac{m_t(A_t-\mu\cot\beta)}{m_{\tilde g}^2} \frac{100\ \text{GeV}}{m_{\tilde g}} \frac{Im[V_{33}^{SKM*} V_{33}^{D_L*} V_{32}^{U_R} V_{cb}^{KM}]}{|V_{cb}^{KM}|^2}, \end{aligned} \tag{14}$$

where $m_\tau = 1.78$ GeV, $m_B = 5.28$ GeV, $m_b \simeq 4.5$ GeV and $m_c \simeq 1.5$ GeV are the masses of the τ^+ lepton, B^+ meson, b and c quarks respectively, p_B and p_D are the four momenta of the B and D mesons, V^{KM} is the KM matrix, and where f_-^B and f_+^B are the form factors (c.f. eq. (6)).

In the heavy quark effective limit with $m_B, m_D \to \infty$, $f_-^B/f_+^B = -(m_B-m_D)/(m_B+m_D) \simeq -0.48$ [25,26]. Since $m_\tau^2 \le (p_B-p_D)^2 \le (m_B-m_D)^2$ and thus $0.05 \le -\frac{f_-^B(p_B-p_D)^2}{f_+^B m_B^2} \le 0.20$, we have neglected this correction in our estimate in eq. (14). As there exists no constraint on $V_{32}^{U_R}$ before we have enough $t \to cZ/\gamma$ events at the Fermilab Tevatron or future high energy colliders, $|V_{32}^{U_R}| \sim \mathcal{O}(1)$ is currently allowed. To estimate the largest possible effect from W exchange, we assume the SUSY mass parameters in eq. (14) to be 100 GeV and maximal squark mixing with $|V_{32}^{U_R}| = 1/\sqrt{2}$. Recall from eq. (5) that I_W^- can be of order 10 for $m_{\tilde t}$ and $m_{\tilde b}$ lighter than $m_{\tilde g}/2$. It is thus seen that $|P_\tau^\perp|$ from W-exchange is no bigger than a few $\times 10^{-3}$.

The charged Higgs exchange contribution to $B^+ \to \overline{D}^0 \tau^+ \nu_\tau$ is dominated by the

diagram involving the $H^- \tilde{t}_R \tilde{b}_L^*$ coupling (see fig. 2 with the external lines properly relabeled). In the B rest frame, for the outgoing τ^+ and ν_τ coming at right angle, the τ transverse polarization is estimated as

$$\begin{aligned} P_\tau^\perp|_{\mathbf{n}_\tau \cdot \mathbf{n}_\nu = \mathbf{0}}^{H^+} &\simeq \frac{\alpha_s}{3\pi} I_{H^+} \frac{\mu + A_t \cot\beta}{m_{\tilde{g}}} \frac{m_t m_\tau}{m_{H^+}^2} \tan\beta \frac{m_B}{m_b - m_c} \frac{|\mathbf{p}_\tau|}{E_\tau} \left(1 + \frac{f_-^B (p_B - p_D)^2}{f_+^B m_B^2}\right) \times \\ &\qquad \times \frac{Im[V_{33}^{H^+ *} V_{33}^{D_L *} V_{32}^{U_R} V_{cb}^{KM}]}{|V_{cb}^{KM}|^2} \\ &\simeq 6 \times 10^{-4} I_{H^+} \frac{|\mathbf{p}_\tau|}{E_\tau} \tan\beta \frac{\mu + A_t \cot\beta}{m_{\tilde{g}}} \frac{(100 \text{ GeV})^2}{m_{H^+}^2} \frac{Im[V_{33}^{H^+ *} V_{33}^{D_L *} V_{32}^{U_R} V_{cb}^{KM}]}{|V_{cb}^{KM}|^2}. \end{aligned} \tag{15}$$

The size of $P_\tau^\perp$ from H^+ exchange depends, among other things, on $V_{32}^{U_R}$, $\tan\beta$, and the charged Higgs mass m_{H^+}. Assuming $m_{H^+} \simeq 100$ GeV, the present limit on $\tan\beta / m_{H^+}$ [19] allows for $\tan\beta$ as large as ~ 50. Models with maximal right-handed up-type squark mixing in the second and third families can be motivated [16] with $|V_{32}^{U_R}| = \sqrt{2}/2$ [18]. Taking $|V_{cb}^{KM}| = 0.04$, $|V_{33}^{H^+}| = |V_{33}^{D_L}| \simeq 1$, and $m_{\tilde{t}} = m_{\tilde{b}} = m_{\tilde{g}} = |\mu|$, we have for the magnitude of $P_\tau^\perp$ in $B^+ \to \overline{D}^0 \tau^+ \nu_\tau$ decay

$$P_\tau^\perp|_{\mathbf{n}_\tau \cdot \mathbf{n}_\nu = \mathbf{0}}^{B^+ \text{ decay}} \leq 4 \times 10^{-1}, \tag{16}$$

where the upper limit in eq. (16) corresponds to the maximally allowed value of $\frac{|\mathbf{p}_\tau|}{E_\tau} \simeq 0.73$.

This limit on $P_\tau^\perp$ is about two orders of magnitude bigger than that of the W exchange contribution, and is larger than the Higgs contribution in the absence of squark mixing by a factor of $\frac{m_t}{m_b |V_{cb}^{KM}|} \simeq 10^3$. At the proposed B factories, about 10^8 B's are collected per year, and with that a precision of $10^{-2} - 10^{-1}$ can be achieved on $P_\tau^\perp$ in the decay $B^+ \to \overline{D}^0 \tau^+ \nu_\tau$. We therefore conclude that future B factories have a promising prospect to detect transverse polarization of the τ due to large $\tan\beta$ and large 2-3 family mixing in the right-handed up-type squark sector.

We now come to the semileptonic D decay, $D^+ \to \overline{K}^0 \mu^+ \nu$. For an estimate of $P_\mu^\perp$ in this decay, we consider the potentially large contribution from charged Higgs exchange involving the $H^- \tilde{t}_R \tilde{b}_L^*$ coupling. In the rest frame of the D meson, $P_\mu^\perp$ for $\mathbf{n}_\mu \cdot \mathbf{n}_\nu = \mathbf{0}$ is given by

$$
\begin{aligned}
P_\mu^\perp|^{H^+}_{\mathbf{n}_\mu\cdot\mathbf{n}_\nu=\mathbf{0}} \simeq & -\frac{\alpha_s}{3\pi} I_{H^+} \frac{\mu + A_t\cot\beta}{m_{\tilde g}} \frac{m_t m_\mu}{m_{H^+}^2} \tan\beta \frac{m_D}{m_c - m_s} \frac{|\mathbf{p}_\mu|}{E_\mu} (1 + \frac{f_-^D (p_D - p_K)^2}{f_+^D m_D^2}) \times \\
& \times \frac{Im[V_{33}^{H^+ *} V_{32}^{D_L *} V_{32}^{U_R} V_{cs}^{KM}]}{|V_{cs}^{KM}|^2} \qquad (17) \\
\simeq & -3\times 10^{-5} I_{H^+} \frac{|\mathbf{p}_\mu|}{E_\mu} \tan\beta \frac{\mu + A_t\cot\beta}{m_{\tilde g}} \frac{(100\ \mathrm{GeV})^2}{m_{H^+}^2} \frac{Im[V_{33}^{H^+ *} V_{32}^{D_L *} V_{32}^{U_R} V_{cs}^{KM}]}{|V_{cs}^{KM}|^2},
\end{aligned}
$$

where $m_D = 1.87$ GeV is the D^+ mass, $f_\pm^D$ are the decay form factors, and p_D and p_K are the four-momenta of the D and K mesons. To estimate the upper limit on $|P_\mu^\perp|$, we take $I_{H^+} = 1$, $\frac{|\mu + A_t \cot\beta|}{m_{\tilde g}} \sim 1$, $m_{H^+} = 100$ GeV and $\tan\beta = 50$. With maximal 2-3 family mixing in both the $\tilde d_{iL}$ and $\tilde u_{jR}$ sectors, $|V_{32}^{D_L}| = |V_{32}^{U_R}| = 1/\sqrt{2}$, we find for the magnitude of $P_\mu^\perp$ in the decay $D^+ \to \overline{K}^0 \mu^+ \nu$,

$$
P_\mu^\perp|^{D^+\ \mathrm{decay}}_{\mathbf{n}_\mu\cdot\mathbf{n}_\nu=\mathbf{0}} < 7\times 10^{-4}, \qquad (18)
$$

where the bound corresponds to the maximal value of $\frac{|\mathbf{p}_\mu|}{E_\mu} \simeq 0.99$. This effect is too small to be seen at the proposed τ-charm factory, even though the upper limit is already larger by a factor of $\sim \frac{m_t}{m_c V_{cs}^{KM}} \sim 10^2$ than the corresponding contribution in the absence of squark family mixing.

It is important to notice that the separate upper limits on $P_l^\perp$ in K, B and D decays given by eqs. (13), (16) and (18), may not be satisfied simultaneously as a consequence of the unitarity constraints on the squark mixing matrices. These decays have different dependence on the squark family mixings, $|P_\mu^\perp| \propto |V_{31}^{U_R} V_{32}^{D_L}|$ for K^+ decay, $|P_\mu^\perp| \propto |V_{32}^{U_R} V_{32}^{D_L}|$ for D^+ decay, and $|P_\tau^\perp| \propto |V_{32}^{U_R} V_{33}^{D_L}|$ for B^+ decay. Therefore, to have large mixing in the 2-3 sector of V^{U_R} would preclude large mixing in the 1-3 sector of V^{U_R}, and vice versa. However, this does not exclude the possibility of other mixing patterns that may allow both $|V_{31}^{U_R}|$ and $|V_{32}^{U_R}|$ to be large. It is thus fair to say that there is no strong correlation among the different $P_l^\perp$'s, and that separate experiments involving both kaon and B factories are required to probe the squark mixing matrices.

The analysis given above has demonstrated several new features of T violation in the presence of squark family mixings. Generally speaking, SUSY contributions to $P_l^\perp$ are

loop suppressed and mechanisms have to be found to overcome this suppression in order that $P_l^\perp$ can be observed in the near future. Misalignment between quark and squark mass eigenstates can lead to a sensitivity to the top mass for kaon, D and B semileptonic decays. If the relevant squark family mixing is not small, this could lead to several orders of magnitude enhancement in $P_l^\perp$ relative to the case where the squark family mixing is absent. We find that in SUSY models with large $\tan\beta$ and large squark family mixings, charged Higgs exchange effects can compensate the loop suppression and give values of $P_\mu^\perp$ on the order of 10^{-3} in $K_{\mu 3}^+$ and $P_\tau^\perp$ of order 10^{-1} in B^+ decay. Both are very exciting prospects for experimental detection at the on-going KEK experiment E246 and the B factories respectively. However, higher precision measurements for both decays are necessary in order to cover moderate squark family mixings and the low $\tan\beta$ region of SUSY parameter space. On the other hand, the transverse muon polarization in D^+ decay is found to be smaller than 10^{-3}, which is too small for detection at the proposed τ-charm factory. Although T violation could also come from slepton family mixing, these effects are expected to be less prominent than from squark mixing because of the weak coupling suppression and m_τ/m_t suppression.

We would like to thank G. Eilam, K. Kiers, Y. Kriplovich and Y. Kuno for discussions, and W.J. Marciano for informative correspondence and comments on the manuscript. This work is partially supported by the Natural Sciences and Engineering Research Council of Canada.

REFERENCES

[1] B.A. Campbell, Phys. Rev. D28 (1983) 209;
M.J. Duncan, Nucl. Phys. B 221 (1983) 285;
J.F. Donoghue, H.P. Nilles, and D. Wyler, Phys. Lett. B 128 (1983) 55;
L.J. Hall, V.A. Kostelecky, and S. Raby, Nucl. Phys. B 267 (1986) 415;
F. Gabbiani and A. Masiero, Nucl. Phys. B 322 (1989) 235;
J.S. Hagelin, S. Kelley, and T. Tanaka, Nucl. Phys. B 415 (1994) 293;
F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, hep-ph/9604387.

[2] S. Bertolini and F. Vissani, Phys. Lett. B 324 (1994) 164;
T. Inui, Y. Mimura, N. Sakai, and T. Sasaki, Nucl. Phys. B 449 (1995) 49;
S.A. Abel, W.N. Cottingham, and I.B. Whittingham, Phys. Lett. B 370 (1996) 106.

[3] S. Dimopoulos and L.J. Hall, Phys. Lett. B 344 (1995) 185;
R. Barbieri, L.J. Hall, and A. Strumia, Nucl. Phys. B 445 (1995) 219;
R. Barbieri, L.J. Hall, and A. Strumia, Nucl. Phys. B 449 (1995) 437;
I.B. Khriplovich and K.N. Zyablyuk, hep-ph/9604211.

[4] N.G. Deshpande, B. Dutta, and E. Keith, Phys. Rev. D 54 (1996) 730; hep-ph/9605386.

[5] J.J. Sakurai, Phys. Rev. 109 (1958) 980;
N. Cabbibo and A. Maksymowicz, Phys. Lett. 9 (1964) 352, (E) *ibid* 11 (1964) 360, (E) *ibid* 14 (1966) 72.

[6] L.B. Okun and I.B. Khriplovich, Yad. Fiz. 6 (1967) 821 [Sov. J. Nucl. Phys. 6 (1968) 598].

[7] M. Leurer, Phys. Rev. Lett. 62 (1989) 1967;
P. Castoldi, J.M. Frere and G. Kane, Phys. Rev. D 39 (1989) 263;
H.Y. Cheng, Phys. Rev. D 28 (1983) 150.

[8] A.R. Zhitnitskii, Yad. Fiz. 31 (1980) 1024 [Sov. J. Nucl. Phys., 31 (1980) 529]

[9] E. Golowich and G. Valencia, Phys. Rev. D 40 (1989) 112.

[10] S. Weinberg, Phys. Rev. Lett. 37 (1976) 657.

[11] H.Y. Cheng, Phys. Rev. D 26 (1982) 143;
R. Garisto and G. Kane, Phys. Rev. D 44 (1991) 2038;
G. Belanger and C.Q. Geng, Phys. Rev. D 44 (1991) 2789.

[12] D. Atwood, G. Eilam and A. Soni, Phys. Rev. Lett. 71 (1993) 492.

[13] R. Garisto, Phys. Rev. D 51 (1995) 1107.

[14] Y. Grossman and Z. Ligeti, Phys. Lett. B 347 (1995) 399.

[15] E. Christova and M. Fabbrichesi, Phys. Lett. B 315 (1993) 113.

[16] M. Dine, R. Leigh, and A. Kagan, Phys. Rev. D 48 (1993) 4269;
Y. Nir and N. Seiberg, Phys. Lett. B 309 (1993) 337;
M. Leurer, Y. Nir, and N. Seiberg, Nucl. Phys. B420 (1994) 468.

[17] N. Arkani-Hamed, H.-C. Cheng, and L.J. Hall, Phys. Rev. D 53 (1996) 413; Phys. Rev. D 54 (1996) 2242.

[18] M.P. Worah, Phys. Rev. D 54 (1996) 2198.

[19] Y. Grossman, H. Haber and Y. Nir, Phys. Lett. B 357 (1995) 630.

[20] R. Barbieri and G.F. Diudice, Phys. Lett. B 309 (1993) 86;
R. Garisto and J.N. Ng, Phys. Lett. B 315 (1993) 372;
Y. Okada, Phys. Lett. B 315 (1993) 119;
N. Oshimo, Nucl.Phys. B404 (1993) 20;
M.A. Diaz, Phys. Lett. B 322 (1994) 207;
F.M. Borzumati, Z.Phys. C63 (1994) 291;
S. Bertolini and F. Vissani, Z. Phys. C 67 (1995) 513.

[21] M.K. Campbell *et al.* Phys. Rev. Lett. 47 (1981) 1032; S.R. Blatt *et al.*, Phys. Rev. D 27 (1983) 1056.

[22] Y. Kuno, Nucl. Phys. B (Proc. Suppl.) 37A (1994) 87;

[23] R. Adair *et al*, muon polarization working group report, hep-ex/9608015.

[24] W.J. Marciano, Private communication.

[25] We thank C. Lee for a discussion on this point.

[26] M. Neubert and V. Rieckert, Nucl. Phys. B 382 (1992) 97.

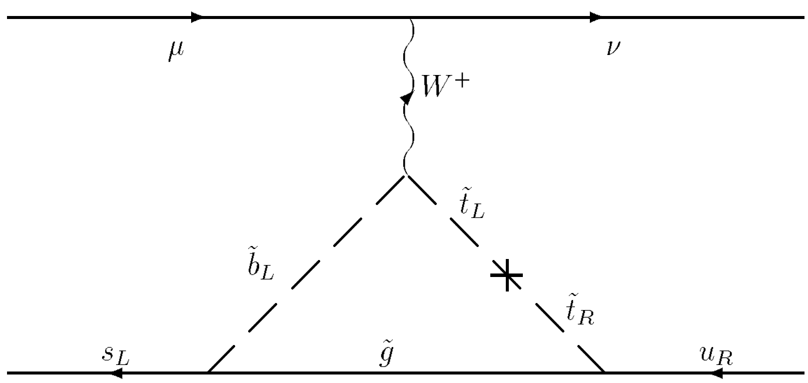

Fig. 1: Quark level contribution to the semileptonic decay of $K^+ \to \pi^0 \mu^+ \nu_\mu$ due to W^+ exchange. The corresponding diagram for $B^+ \to \overline{D}^0 \tau^+ \nu_\tau$ can be obtained by replacing s_L and u_R with b_L and c_R, and by relabeling the external lepton lines.

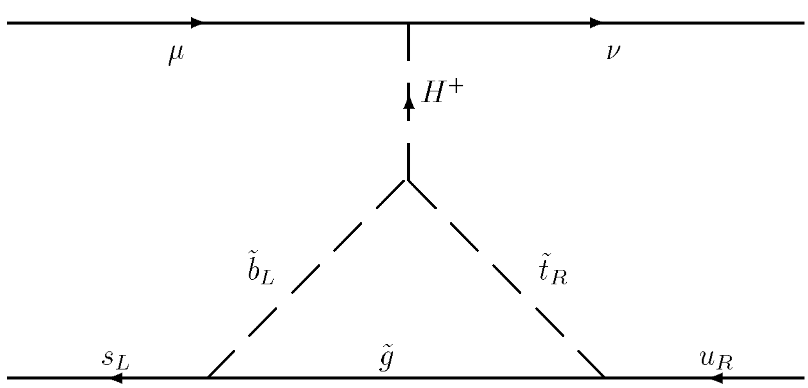

Fig. 2: Same as Fig. 1, the dominant diagram due to H^+ exchange.